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Sensitivity of the eigenfunctions and the level curvature distribution in quantum billiards

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Abstract. In searching for the manifestations of sensitivity of the eigenfunctions in quantum billiards (with Dirichlet boundary conditions) with respect to the boundary data (the normal derivative) we have performed instead various numerical tests for the Robnik billiard (quadratic conformal map of the unit disk) for 600 shape parameter values, where we look at the sensitivity of the energy levels with respect to the shape parameter. We show the energy level flow diagrams for three stretches of fifty consecutive (odd) eigenstates each with index 1,000 to 2,000. In particular, we have calculated the (unfolded and normalized) level curvature distribution and found that it continuously changes from a delta distribution for the integrable case (circle) to a broad distribution in the classically ergodic regime. For some shape parameters the agreement with the GOE von Oppen formula is very good, whereas we have also cases where the deviation from GOE is significant and of physical origin. In the intermediate case of mixed classical dynamics we have a semiclassical formula in the spirit of the Berry-Robnik (1984) surmise. Here the agreement with theory is not good, partially due to the localization phenomena which are expected to disappear in the semiclassical limit. We stress that even for classically ergodic systems there is no global universality for the curvature distribution, not even in the semiclassical limit.

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1 Introduction

The main motive of our present work is to understand and analyze the notion of *sensitivity of the eigenfunctions* of classically chaotic quantal Hamiltonian systems with respect to boundary conditions, and/or boundary data and/or the system parameter. It is expected that such sensitivity would indeed correlate with classical chaos whereas in the classically integrable systems it would be lacking. It was the important pioneering idea by Percival (1973), based on the semiclassical thinking, that proposed to classify the eigenstates and energy levels in regular and irregular depending on whether they are associated with (supported by) classical regular regions (invariant tori) or by classical chaotic regions in the classical phase space. This picture is in fact the basis of the Berry-Robnik (1984) approach to describe the statistical properties of energy spectra in the transition region between integrability and full chaos (ergodicity), which has been recently fully confirmed (Prosen and Robnik 1994a,b). (The role of localization phenomena on chaotic components implying the fractional power law level repulsion and a Brody-like behaviour has been understood and demonstrated as well, showing that in the strict semiclassical limit, where the localization disappears, the statistics converges indeed to Berry-Robnik (1984).) Moreover, we have recently performed the first *dynamical* separation of regular and irregular levels and eigenstates (Li and Robnik 1994c, 1995a,b), thereby explicitly verifying the ingredients in the Berry-Robnik picture, in particular the Principle of Uniform Semiclassical Condensation (PUSC) (Li and Robnik 1994a, Robnik 1988, 1993).

It was already Percival's idea (1973) to look at the second differences of the energy levels with respect to some system parameter as an indication of strong sensitivity (Pomphrey 1974), which is the precursor of the level curvature concept (Gaspard *et al* 1989, 1990). Of course, in an *unfolded* (see e.g. Bohigas 1991) energy level flow we have stationarity in the mean so that not only the mean "velocity" but also the average level curvature is zero, the important point being that the curvature distribution becomes broader and in fact has some *universal features* in the tail as predicted by Gaspard *et al* (1989,1990). Even at all values of the curvature, after an appropriate normalization of the curvature, this distribution for classical ergodic systems can - but need not - be close to the prediction of the random matrix theory (RMT) whose functional form has been conjectured by Zakrzewski and

Delande (1993) and proven by von Oppen (1994,1995). Thus the curvature of individual levels is of course not indicative of chaos at all but it is its distribution for the ensemble of levels which matters. We should emphasize, however, that for reasons explained in section 3 (noninvariance with respect to re-parametrization) globally the level curvature distribution (after unfolding and after normalization) *need not* be universal even for the class of fully chaotic (ergodic) dynamical systems. Therefore, the most important aspect here is just the broadness of the curvature distribution whose dispersion is of order unity. In the opposite extreme of integrable systems we would find just delta spike distribution for level curvatures, simply because the levels do not interact at all (degeneracies and level crossings are allowed) so that locally after unfolding the level flow is just straight lines. In this paper we address also the intermediate case of mixed classical dynamics and show that upon the assumptions involved in the Berry-Robnik picture mentioned before it can be modelled very easily, but its statistically significant numerical demonstration is quite hard to achieve, for which the present work is the first step in this direction.

When starting this project we were thinking quite generally about the notion of sensitivity of eigenstates introduced above and tried to implement some of these ideas also numerically in the sense of looking at the norm of the second derivatives of the wavefunctions, but we had to abandon this analysis simply because it is numerically too massive even for the lowest states. Therefore as a partial substitute we looked very carefully and in detail at the energy level flow and the curvature distribution. In section 2 we define the system and the technique and show the energy level flow. In section 3 we perform the analysis of the curvature distribution in the regime of full chaos (ergodicity), in section 4 we study the curvature distribution in the intermediate case of mixed classical dynamics, and in section 5 we conclude and discuss the main results.

2 The billiard system, technique and energy level flow

Our billiard system that we use to study the sensitivity of the eigenstates is defined as the quadratic (complex) conformal map $w = z + \lambda z^2$ from the unit disk $|z| \leq 1$ from the z plane onto the w complex plane. The system has been introduced by Robnik (1983) and further studied by Hayli *et al* (1987), Frisk (1990), Bruus and Stone (1994) and Stone and Bruus (1993a,b) for various parameter values λ. Since the billiard (usually called Robnik billiard) has analytic boundary it goes continuously from integrable case (circle, $\lambda = 0$) through a KAM-like regime of small $\lambda \leq 1/4$ with mixed classical dynamics, becomes nonconvex at $\lambda = 1/4$ (the bounce map becomes discontinuous), where the Lazutkin caustics (invariant tori) are destroyed giving way to potential ergodicity. As shown by Robnik (1983) the classical dynamics at these values of λ is predominantly chaotic (almost ergodic), although Hayli *et al* (1987) have shown that there are still some stable periodic orbits surrounded by very tiny stability islands up to $\lambda = 0.2791$. At larger λ we have reason and numerical evidence (Li and Robnik 1994b) to expect that the dynamics can be ergodic. It has been recently rigorously proven by Markarian (1993) that for $\lambda = 1/2$ (cardioid billiard) the system is indeed ergodic, mixing and K. This was a further motivation to study the cardioid billiard classically, semiclassically and quantanlly by several groups e.g. by Bäcker *et al* (1994) and Bäcker (1995), Bruus and Whelan (1995).

The numerical technique to solve the quantum billiard at various values of the shape parameter $\lambda \in [0, 1/2]$ is the conformal mapping diagonalization technique devised by Robnik (1984) and further improved by Prosen and Robnik (1993,1994b) with which, using Convex C3860 machine, we can at best obtain about 12,000 good levels (in the sense of having an accuracy of at least one percent of the mean level spacing) at each λ and symmetry (parity) class. Our calculations have been done only for odd parity states for about 600 values of λ such that the lowest 3,000 states have an accuracy of at least 10^{-3} of the mean level spacing. (For technical remarks see (Prosen and Robnik 1993); in the present case the dimensionality of the matrices that we diagonalized was always 11,125.) In order to collect all these 600 times 3,000 energy levels we have used 75 days of CPU time.

In figures 1-3 we show the energy level flow for fifty consecutive eigenstates for the entire interval of $\lambda \in [0, 1/2]$, evaluated (numerically calculated) at 600 values of λ which is sufficiently dense to make the appearance of the flow seemingly continuous. We must emphasize that the plotted energy E here is in fact the *unfolded* energy after solving the Helmholtz equation $\Delta\psi + \tilde{E}\psi = 0$ with Dirichlet boundary condition (vanishing ψ on the boundary $|z| = 1$) for the wavefunction ψ corresponding to the pre-unfolded energy $\tilde{E}$, by using the well known Weyl formula with perimeter, curvature and corner corrections, given e.g. in Prosen and Robnik (1993) equation (18). In figure 1 we show states from 1,000 to 1,050, then in figure 2 the eigenstates from 1,500 to 1,550 and in figure 3 the energy levels from 2,000 to 2,050. In each of these cases we also show some fine structure of the energy spectrum in two magnified insets. The main observation is of course that due to unfolding the level flow is indeed stationary, that is the mean "velocity" vanishes, i.e. $< dE_n/d\lambda >= 0$. At sufficiently small λ like e.g. $\lambda \leq 0.1$ the level flow is laminar and all levels are very flat in this KAM-like regime. This region is then followed by intermediate values of λ between 0.15 and 0.43 where classically we have quite strong chaos implying strong level repulsion giving rise to the turbulent level flow exhibiting many avoided crossings. Finally, at large λ, say $\lambda \geq 0.43$, we have classically very strong chaos (in fact ergodicity, mixing and K property) which paradoxically gives rise to such a strong level repulsion that we again find laminar flow. Unlike in the nearly integrable regime of small λ here with increasing energy we would eventually recover the fully turbulent flow typically associated with chaos. The statistical properties of the energy spectra in this regime in the strict semiclassical limit obey the universal laws predicted by GOE of random matrix theories (e.g. Bohigas 1991). This has been studied in detail by Prosen and Robnik (1993,1994a,b) and by Li and Robnik (1994c,1995b). As for the curvature distribution it is possibly statistically satisfactorily described by the random matrix theories (Zakrzewski and Delande 1993, Delande and Zakrzewski 1994, von Oppen 1994,1995), certainly in the tails of the curvature distribution, but otherwise we have no reason to expect universality of the curvature distribution for all values of the curvature.

3 Level curvature distribution in fully chaotic systems

In order to uncover the universal features in the curvature distribution of the energy level flow, as predicted by Gaspard *et al* (1989,1990) for the tails of the curvature distribution, we have to appropriately normalize the curvature in such a manner that the normalized curvature is measured in certain natural units. The answer is well known (Zakrzewski and Delande 1993, Gaspard *et al* 1990), namely

$$k = \frac{K}{\pi\beta\rho < (dE_n/d\lambda)^2 >}, \tag{1}$$

where $K = d^2E_n/d\lambda^2$ is the actual curvature of the n-th energy level with the eigenvalue E_n, i.e. its second derivative, β is a constant which according to the RMT is equal to 1,2 and 4 for GOE, GUE and GSE, respectively, provided the classical dynamics of the system is completely chaotic (ergodic). In dynamical systems we use $\beta = 1$ if the system has antiunitary symmetry, $\beta = 2$ if the system has no such symmetry and in case that the system has half integer spin, an antiunitary symmetry but no rotational symmetry we take $\beta = 4$ (Berry and Robnik 1986, Robnik and Berry 1986, Robnik 1986). ρ is the local density of energy levels, which after unfolding is by construction equal to unity, and therefore the mean value of all the derivatives of E_n with respect to λ vanish, in particular the average "velocity" is zero, i.e. $< dE_n/d\lambda >= 0$. $< (dE_n/d\lambda)^2 >$ is the average of the squared "velocity" of the energy levels taken over a suitable ensemble of consecutive energy levels (the spectral stretch). It can be easily verified that k thus defined is dependent on the parametrization of the system and of its energy spectrum. Namely, after re-parametrization $\mu = \mu(\lambda)$, we obtain

$$k_\mu = k_\lambda - \frac{v_\lambda}{\pi\beta\rho < v_\lambda^2 >}\frac{\mu''}{\mu'}, \tag{2}$$

where k_λ, v_λ are the (normalized) curvature and the velocity calculated with respect to parameter λ, k_μ is the (normalized) curvature in μ-parametrization calculated according to (1), μ'' and μ' are the second and first derivative of μ with respect to λ, whereas β and ρ are as previously defined, and actually after unfolding $\rho = 1$.

The first immediate conclusion following this curvature re-parametrization equation is that *as for the global curvature distribution there cannot be any universality simply because the distribution of the normalized curvature depends on parametrization.* There are the following important remarks. The curvature distribution *is* invariant with respect to the linear transformations $\mu = const \times \lambda$. Knowing that typically the velocity distribution is Gaussian[3] and thus rapidly decaying with v (Delande and Zakrzewski 1994), whilst the curvature distribution typically has algebraic tails, we can see from equation (2) that *the tails* of the curvature distribution can be universal, and they have been predicted for the first time by Gaspard *et al* (1989,1990). This universal feature is correctly captured by the random matrix model as conjectured by Zakrzewski and Delande (1993) and proven by von Oppen (1994, 1995), namely

$$P(k) = \frac{C_\beta}{(1+k^2)^{(\beta+2)/2}}, \tag{3}$$

where $\beta = 1, 2, 4$ for GOE, GUE and GSE, respectively, and C_β is the normalization constant. We should emphasize that universality at all k cannot exist and therefore the random matrix model (3) does not have the status of some universal law, but is in fact just a case study, so that there remains the open question to clarify under what conditions it can be expected to model the curvature distribution of a (one parameter) family of classically ergodic Hamiltonian systems. There are cases, probably somewhat accidental, where the agreement with RMT is very good at all k and we shall report on such a case below.

The first example that we give is the billiard system at $\lambda = 0.41$, where the classical dynamics is fully chaotic (probably ergodic, mixing and K) and we have calculated the curvatures for 1,000 consecutive energy levels from the 2,001-st to the 3,000-th eigenstate. The result is shown in figure 4b, where we show the cumulative curvature distribution

$$W(k) = \int_{-\infty}^{k} P(t)dt, \tag{4}$$

[3]We have analyzed also the velocity distribution for many parameter values λ and in all cases the tails of the distribution are consistent with the Gaussian distribution, and where the statistical significance is sufficiently high we have also observed agreement for all velocities.

in comparison with the GOE von Oppen formula based on (3). For illustrative purposes we also show the histogram in figure 4a in comparison with the von Oppen formula ($\beta = 1$). The agreement is seen to be very good for reasons that still have to be understood. Namely, there cannot be any global universality and therefore there is no a priori reason that RMT model should describe the dynamical systems' curvatures. Interestingly, if we take the same number of consecutive levels but at lower energies, namely from the 201-st to the 1,200-th eigenstate, we observe a substantial degradation of the agreement mainly manifested in the pronounced central peak at small k as shown in figures 5a,b. Thus in this case in the semiclassical limit $E_n \to \infty$ the GOE result seems to apply and the deviations from the von Oppen formula seem to be attributed to low energies, i.e. not sufficiently small effective $\hbar$.

We have checked the curvature distributions for at least one hundred values of λ covering the fully chaotic region at $\lambda \geq 0.2791$ and the transition region (KAM-like regime) of small λ. In all cases of full chaos (ergodicity) the agreement with von Oppen formula was much worse than in figure 4a,b even at high eigenstates and this is demonstrated collectively in figure 6a,b where we show the curvature distributions for 34 values of λ, at each of them taking the spectral stretch between the 2,001-st and the 3,000-th eigenstate. The range of λ is $[0.27, 0.435]$ at equal steps $\Delta\lambda = 0.005$. The histogram in figure 6a gives an impression that the agreement with GOE formula of von Oppen is very good, but the much more informative cumulative plot in figure 6b reveals that the data are statistically significantly settled (almost negligible fluctuations) and seem to converge to a smooth distribution which, however, notably differs from GOE distribution. It has to be verified whether this disagreement has a dynamical reason and persists or even becomes stronger in the strict semiclassical limit, or else it disappears in the strict semiclassical limit and by some non-understood mechanism agrees with GOE. We think that this latter option is unlikely, since universality in the large for curvature distribution does not exist.

As explained before the curvature distribution can have some universal features and one of them is the algebraic tail predicted by Gaspard *et al* (1989,1990) and captured by von Oppen formula (3). We have explicitly tested this aspect and the results are shown in figure 7 for the systems of figure 6a,b: The agree-

ment as seen in the histogram of figure 7a seems to be very good, however if we instead plot the data cumulatively, namely as a plot of $\log(1 - W(|k|))$ versus $\log(|k|)$, where we have no binning and the numerical accuracy is fully respected, we must conclude that the agreement is poor. In figure 8 we show the tail of the distribution in figure 4a,b and as expected the agreement with theory here is confirmed. Of course the very last part of the tail is statistically not significant since there we have very small number of objects and the expected statistical errors become huge. One of the worst cases that we found for the curvature distribution is at $\lambda = 0.49$, which is deep in the classically ergodic regime, for the eigenstates 2,001 through 5,000, where the distribution is extremely flat as shown in figure 9a,b, but its tails nevertheless obey the theoretical prediction satisfactorily as shown in figure 10, where in the range of $3 \leq |k| \leq 10$ the slope agrees with the RMT slope, namely -3 for $P(k)$ and -2 for $1 - W(k)$. As explained in the previous section the curvature distribution in figure 9a is very flat paradoxically just due to the strong energy level repulsion which gives rise to the laminar level flow (figures 1-3). Certainly, at higher energies we would expect better agreement with the theoretical tails than seen in figure 10a,b.

4 Energy level curvature distribution in transition region between integrability and chaos

Now we turn to the problem of level curvatures of systems in the transition region between integrability and full chaos, with mixed classical dynamics. To our knowledge this problem has not been addressed so far in the literature. But the problem is quite simple in the strict semiclassical limit. We adopt the same assumptions that underly the Berry-Robnik approach (1984), especially the validity of PUSC and the statistical independence of the energy level subsequences associated with the regular components ($j = 1$), and the irregular components ($j = 2, 3, 4, ..., N$) (ordered in decreasing size). These assumptions immediately imply that the curvature distribution $P(k)$ for the entire spectrum is given by the simple additivity formula

$$P(k) = \sum_{j=1}^{N} \rho_j P_j(k), \tag{5}$$

where ρ_j is the fractional phase space volume of the given regular ($j = 1$) components (lumped together in one subsequence), or of the irregular components ($j = 2, 3, 4, ..., N$). Here $P_1(k)$ is the curvature distribution of an integrable system and is just a delta function $\delta(k)$, whereas $P_j(k)$ is the curvature distribution for the spectral subsequence associated with the j-th chaotic component, for which there is no universal formula but under certain not yet understood conditions the von Oppen formula of RMT might be a good model. Its tail, however, must obey the universal prediction of Gaspard *et al* (1989, 1990) captured also in (3), in the strict semiclassical limit.

In the almost integrable KAM-like regime at $\lambda = 0.1$, figure 11a,b, where the classical ρ_1 is estimated $\rho_1 = 0.88$ (Prosen and Robnik 1993), we find strong delta spike with the smooth background modelled by $\rho_2 P_{\rm GOE}(k)$, where $\rho_2 = 1 - \rho_1 = 0.12$ and $P_{\rm GOE}(k)$ is the GOE von Oppen formula (3). Our judgement is that the agreement is satisfactory, given the fact that we have only 1,000 objects, the curvatures of the eigenstates from the 2,001-st through the 3,000-th. Thus the agreement with two-component formula (5) ($N = 2$) is reasonable. As λ is increased to $\lambda = 0.15$ the area of the chaotic components increases to $\rho_2 = 0.64$, $\rho_1 = 0.36$, we observe the drop of the strength of the delta function as shown in figure 12a,b for relatively low states (2,001-3,000). If we go higher in the semiclassical limit by considering the eigenstates 5,001-7,000 we find considerable improvement of the agreement with the semiclassical formula of von Oppen (5), by using the classical value $\rho_1 = 0.36$ for the dashed smooth background, which seems to be qualitatively well captured in figure 13a. Nevertheless, the best fitting procedure to determine the parameter ρ_1 yields $\rho_1 = 0.11$, which is at variance with the classical value $\rho_1 = 0.36$. By going higher in the semiclassical limit we certainly expect further improvement, although the regime where Berry-Robnik assumptions are satisfied is usually very hard to reach (Prosen and Robnik 1993,1994a,b). Therefore we should consider the results of figure 13a,b as the right trend towards the semiclassical formula (5). This is also qualitatively confirmed in figure 14a,b where $\lambda = 0.175$ and the classical $\rho_1 = 0.17$.

In our final plot we attempt to find the best fit of the semiclassical formula (5) with just two components, $j = 1, 2$, and one parameter ρ_1, to the numerical data (cumulative curvature distributions $W(k)$) for fifty values of λ covering the whole range of the mixed dynamics $0 \leq \lambda \leq 1/4$. We plot the quantity D

which is the supremum norm of the difference between the numerical $W(k)$ and the best fit $W(k)$. Unfortunately the significance of our semiclassical fits is not very striking, but it is our impression that the behaviour has the right trend towards Berry-Robnik surmise which underlies the semiclassical formula (5). One indication for this is the calculation of the curvatures for $\lambda = 0.15$ for twice higher energies, namely the eigenstates 5,001-7,000, where the agreement becomes indeed better and D becomes almost twice smaller (indicated by the arrow and the star symbol). One should observe that the agreement is better at larger λ's and smaller ρ_1. In the range of classically almost or completely ergodic motion $0.2791 \leq \lambda \leq 1/2$ we have used not the semiclassical formula (5) but assumed instead $\rho_1 = 0$ and thus used the von Oppen GOE formula (3) with $\beta = 1$. Here for not too large λ the agreement is quite good and somewhere even excellent like $\lambda = 0.41$, as already shown in figure 4a,b and the tails in figure 8a,b. In fact the agreement is very good in the range of $0.27 \leq \lambda \leq 0.44$. For reasons explained already in sections 2 and 3 for λ close to $1/2$ the agreement with GOE becomes quite poor because of the flatness of the curvature distribution which is predicted to gradually disappear when we go to higher energies and thus to smaller effective $\hbar$, i.e. deeper in the semiclassical limit. To demonstrate this we have calculated the curvatures at $\lambda = 0.49$ for twice larger energies in the range of eigenstates 4,001-5,000 resulting in the significantly smaller D as indicated by the arrow and by the star symbol.

5 Discussion and conclusions

We believe that in the present paper we provide further evidence that unlike the spectral fluctuations and their statistics the curvature distribution (defined by some local one parameter family of Hamiltonians) need not obey any universal law such as e.g. the von Oppen formula (3) for the random matrix models, even if the system is fully chaotic (ergodic), and even if we are sufficiently far in the semiclassical limit. One theoretical reason is the lack of the re-parametrization invariance of the normalized curvature k, explicitly demonstrated by equation (2). On the other hand since the "velocities" (of the level flow) typically are Gaussian distributed, the same equation shows that asymptotically at sufficiently large k we can have universality describing the algebraic tail of the curvature distribution as predicted by Gaspard *et al*

(1989,1990) and then correctly reproduced by von Oppen's formula (3). Even being aware of these limitations regarding the universality aspects of the curvature distribution we nevertheless find some examples where the agreement with von Oppen's formula should be judged as very good even for all k. It still has to be understood under what conditions precisely the RMT models would apply to dynamical systems. The substantial (statistically highly significant) deviations from GOE formula have been reported and emphasized already by Zakrzewski and Delande (1993) for the case of the stadium and the hydrogen atom in strong magnetic field and the nonuniversality aspects have been discussed also by Takami and Hasegawa (1992,1994) although for relatively low states and statistically still to be improved. Both groups proposed that this is to be associated with the scarring and other localization phenomena in the eigenstates. It is precisely these phenomena which imply the deviation of spectral statistics from the Berry-Robnik surmise (1984) in the mixed systems and manifest themselves in fractional power law level repulsion and Brody-like behaviour as explained in detail in (Prosen and Robnik 1994a,b, Li and Robnik 1995b). These phenomena disappear in the semiclassical limit where we find uniformly extended chaotic states and the PUSC is fulfilled then, and so is the Berry-Robnik (1984) surmise. However, unlike the spectral statistics the curvature distribution need not converge to the random matrix models not even in the strict semiclassical limit where the localization phenomena disappear. It is thus still an important open theoretical problem, especially in the semiclassical level, to understand under what conditions the random matrix models would apply to the curvature distribution of the quantal dynamical systems. As explained in section 3, equation (2), the global universality of the curvature distribution could show up only in cases where the natural parametrization is somehow restricted to the class of linear transformations.

The sensitivity of the eigenstates (eigenenergies and wavefunctions) on the boundary data, of which one aspect is also the dependence of the eigenstates on the billiard shape parameter, is an important theoretical problem. If such sensitivity correlates with classical chaotic dynamics and at the same time manifests itself in the accuracy of the purely quantal numerical methods, then such a behaviour would be one important manifestation of quantum chaos. We have recently demonstrated (Li and Robnik 1995d) that this is precisely the case when applying the plane wave decomposition method of

Heller (1991), whilst e.g. in the boundary integral method such correlation does not show up (Li and Robnik 1995c). Our present work where we analyze the curvature distribution as a function of the billiard shape parameter is only the first step in direction of this research.

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Figure captions

Figure 1: The unfolded energy level flow of the Robnik billiard for the odd eigenstates from 1,000 to 1,050 (top). The fine structures in the two small windows are magnified and plotted in the bottom left and right boxes, respectively.

Figure 2: The same as figure 1 but for the eigenstates from 1,500 to 1,550.

Figure 3: The same as figure 1 but for the eigenstates from 2,000 to 2,050.

Figure 4: The histogram of the curvature distribution $P(k)$ (a) and the cumulative curvature distribution $W(k)$ (b) for the Robnik billiard at $\lambda = 0.41$. The curvatures are calculated for 1,000 consecutive eigenstates, namely from 2,001 to 3,000. The numerical results (solid line) are compared with the GOE distribution of equation (3), with $\beta = 1$ (dashed line). The agreement in (b) seems to be surprisingly good.

Figure 5: The same as figure 4 but at lower eigenenergies, namely from eigenstates 201 to 1,200. Please note the enhanced peak around $k = 0$ in curvature distribution $P(k)$ (a) and the deviation from GOE distribution in $W(k)$ (b).

Figure 6: The curvature distribution for 34 values of λ in the range of $[0.27, 0.435]$ at equal steps $\Delta\lambda = 0.005$. At each λ the curvatures are calculated for the eigenstates 2,001 through 3,000. The $P(k)$ and $W(k)$ are shown in (a) and (b), respectively. The numerical curve seems to be statistically settled (very small statistical fluctuations giving rise to an apparently smooth step function) but the deviation from the GOE formula of equation (3) as seen in the cumulative distribution $W(k)$ seems to be a physical effect.

Figure 7: The same data as in figure 6 but the $log(P(|k|)$ versus $log(|k|)$ plot (a), and $log(1 - W(|k|))$ versus $log(|k|)$ plot (b). We show these plots in

order to clearly display the behaviour in the tail of the curvature distribution at large $|k|$.

Figure 8: The same data as in figure 4 but the $log(P(|k|)$ versus $log(|k|)$ plot (a), and $log(1-W(|k|))$ versus $log(|k|)$ plot (b). We show these plots in order to clearly display the behaviour in the tail of the curvature distribution at large $|k|$.

Figure 9: The histogram of the curvature distribution $P(k)$ (a) and the cumulative curvature distribution $W(k)$ (b) for $\lambda = 0.49$. The system is already completely chaotic but the deviation from GOE distribution is very big. In this figure 3,000 consecutive eigenstates, namely 2,001 to 5,000, are used to calculate the curvatures.

Figure 10: The same data as in figure 9 but the $log(P(|k|)$ versus $log(|k|)$ plot (a), and $log(1-W(|k|))$ versus $log(|k|)$ plot (b). We show these plots in order to clearly display the behaviour in the tail of the curvature distribution at large $|k|$. The slope of the tail seems to be in better agreement with RMT model than the global $P(k)$ of figure 9.

Figure 11: The histogram of the curvature distribution $P(k)$ (a) and the cumulative curvature distribution $W(k)$ (b) in transition region at $\lambda = 0.1$. In (a) we have the data in full line in comparison with $\rho_2 P_{\rm GOE}(k)$ (dashed), where $\rho_2 = 0.12$ is determined by the classical dynamics. In (b) we plot the same data but cumulatively in comparison with the theoretical cumulative distribution $\rho_1\Theta(k) + \rho_2 P_{\rm GOE}(k)$. The data are the curvatures calculated for the eigenstates 2,001 through 3,000.

Figure 12: The same as figure 11 but for $\lambda = 0.15$ and classical $\rho_1 = 0.36$.

Figure 13: The same as figure 12 but the curvatures are taken from higher eigenergies, namely from 5,001 through 7,000. The agreement with the semi-classical formula in equation (5) is definitely better than for the lower states of figure 12.

Figure 14: The same as figure 11 but for $\lambda = 0.175$ and the classical $\rho_1 = 0.17$.

Figure 15: The quantity D, defined as the supremum norm of the difference of the numerical $W(k)$ and the theoretical $W(k)$, versus λ. In the range of $0 < \lambda \leq 1/4$, D is calculated by comparison with the the best fit $W(k)$ of the semiclassical two-component formula of equation (5). For $\lambda > 1/4$, D is calculated by comparison with the GOE von Oppen formula (3). The $\diamond$ is the result for the energy stretch of eigenstates 2,001 through 3,000, while $\star$ at $\lambda = 0.49$ represents the data from eigenstates 4,001 to 5,000 and $\star$ at $\lambda = 0.15$ represents the data from 5,001 to 7,000.